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UTILITY OF DUGDALE SOLUTION FOR TWO UNEQUALCRACKS IN AN INFINITE PLATE

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INTRODUCTION

The collinear crack problems are significant from the view point of its occurrence in different life conditions, such as rock media, defects occurring in course of manufacturing process. Hence, these problems are required to be studied.

Dugdale (1960) proposed a simple model when a straight slit weakening a plate is opened in mode – I type deformation under tensile force applied remotely. The rims of the plastic zones formed were takento be closed by normal uniform constant yield point stress. This model was modified by Harrop when rims of the plastic zones were closed by cohesive normal yield point stress distribution, varying as an arbitrary second order polynomial. Theocaris found Dugdale model solution for two collinear straight cracks. We propose a crack closer model for a circular arc cracks weakening plates.

An analytic solution for two equal length collinear strip yield cracks weakening an unbounded plate has been obtained by Collins and Cartwright (2001) using complex variable technique developed by Muskhelishvilli (1953). The problem was further extended for the case of two collinear unequal straight cracks weakening an infinite sheet. An analytic solution was obtained for infinite elastic perfectly - plastic plate with center crack loaded by two pairs of point tensile force.

Present paper obtains a modified Dugdale model solution for two unequal homogeneous elastic perfectly
– plastic plate weakened by two unequal hairline collinear straight cracks.

Material and method

According to Muskhelishvilli’s complex variable formulation, the stress components Pij (i, i = x,y) may be expressed in terms of two complex potentials Φ (z) and Ω (z) as

$$P_{yy} - i P_{xy} = \Phi(z) + \Omega(z) - (z - z)\,\Phi(\bar{z})$$

Consider an unbounded plate weakened by unequal collinear straight cracks Li (i=1, 2, 3, .n) with end points ai, bi. The configuration so obtained is subjected to the following boundary conditions:

a) Pyy act along the rims of the crack Li (i=1,2,3,….n)

b) No stresses act at infinite boundary of the plate.

c) Displacements are single-valued around the rims of cracks.

Boundary conditions together with above equation yields two Hilbert problems under the assumptionlim y tend to zero.

Where

$$X(z) = \prod_{K=1}^{n} (z-a_k)^{1/2} (z-b_k)^{1/2}, \; L = ULi,$$

$$2\,p(t) = (P^+_{yy} + P^-_{yy}) - i\,(P^+_{xy} + P^-_{xy})\,; \quad 2q(t) = (P^+_{yy} - P^-_{yy}) - i(P^+_{xy} - P^-_{xy})$$

and

$$P_n(z) = C_0 z^n + C_1 z^{n-1} + \ldots + C_n$$

Arbitrary constants Ci (i= 1, 2, 3, n) are determined using boundary conditions (b) and (c), together with single-valuedness condition of displacement, taken from Muskhelishili (1953)

$$2(k+1)\int_L \underline{P_n(t)}\,dt + k\int_L [\Phi^+(t) - \Phi^-(t)]\,dt + \int_L [\Omega^+(t) - \Omega^-(t)]dt = 0.$$

Statement of the problem

Let us assume an infinite homogeneous elastic perfectly-plastic plate to occupy *xoy*-plane. Let the twointervals be [d1, c1] and [b1, a1], on *ox-axis*, respectively.

Then, let a uniform constant tension be applied at the infinite boundary in a direction perpendicular to the rims of the cracks. This results in the opening of the cracks in Mode-I type deformations forming plastic zones. Rims of these zones are subjected to the stress P± yy = t 2σye, P± xy = 0, where σye denotes yield point stress of the plate and t is any point on any of the plastic zone. *(Refer figure1)*

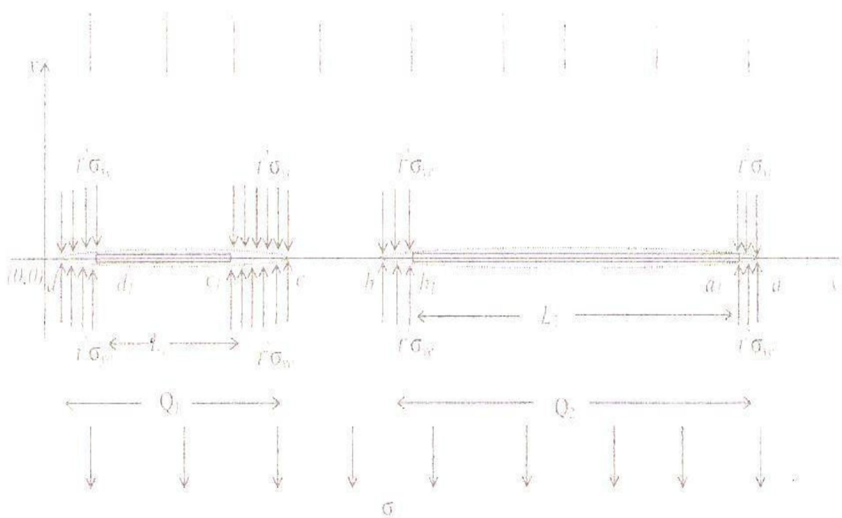

Figure 1. Schematic representation of the configuration of the problem.

Solution of the problem

The above problem is solved using principle of superposition of the stress intensity factors for two component problems contributing towards the stress singularity.

An infinite homogeneous elastic perfectly-plastic plate is weakened by two unequal hairline collinear straight cracks Q_1 and Q_2. The crack: Q_1 (= $\Gamma_1 U L_1 U \Gamma_2$) occupies the interval [d,c] and crack Q_2 (= $\Gamma_3 U\ \Gamma_2 U \Gamma_4$) occupies the interval [b, a] along *ox*-axis. Boundary conditions of the problems are

- Rims of the cracks Q_1 and Q_2 are stress free
- Infinite boundary of the plate is subjected to stress distribution $P_{yy}^{\pm} = \sigma_\infty$, $P_{xy}^{\pm} = 0.$
- Displacements are single-valued around the rims of cracks Q_1 and Q_2

Using boundary conditions i) and iii), the desired potential $\Phi^I(z)$ may be written as

$$\Phi^I(z) = \frac{C_0 z^2 + zC_1 + C_2}{X(z)} - \frac{\sigma_\infty}{4}$$

$$X(z) = \sqrt{z-a}\,\sqrt{z-b}\,\sqrt{z-c}\,\sqrt{z-d}$$

$W_0 = gV_0$, $W_1 = g[aV_0 + (d - a)V_1]$, $W_2 = g[a_2V_0 + 2a(d - a)\, V_1 + (d - a)^2\, V_2]$

$V_0 = F(k)$, $V_1 = \Pi(\alpha^2, k)$,

$$V_2 = \frac{1}{2(\alpha^2 - 1)(k^2 - \alpha^2)}\,[\,\alpha^2 E(k) + (k^2 - \alpha^2) F(k) + (2k^2\alpha^2 + 2\alpha^2 - \alpha^4 - 3k^2)\,\Pi(\alpha^2, k)]$$

$$g = \frac{2}{\sqrt{a-c}\,\sqrt{b-d}} \qquad \alpha = \frac{d-c}{a-c} < 0, \qquad k = \frac{(a-b)(c-d)}{(a-c)(b-d)}$$

$$Y_2 = g[c^2U_0 + 2c(b-c)\,U_1 + (b-c)\,U_2, \; Y_1 = g[cU + (b-c)U_1], \; Y_0 = gU_0.$$

$$\beta^2 = \frac{a-b}{a-c} < 1, \quad \varphi = \frac{\pi}{2}, \quad U_0 = F(k), \quad U_1 = \Pi(\beta^2, k),$$

$$U_2 = \frac{1}{2(\beta^2 - 1)(k^2 - \beta^2)}\,[\beta^2 E(k) + (k^2 - \beta^2) F(k) + (2k^2\beta^2 + 2\beta^2 - \beta^4 - 3k^2)\,\Pi(\beta^2, k)]$$

and $F(k)$, $\Pi(\alpha^2,k)$ are the complete elliptic integral of first and third kind, respectively.

CONCLUSION

The investigations are carried out for different crack length ratios, for example, for the case of two cracks being of equal in length. For this case when crack lengths are equal, the results match with the results of Theocaris (1983).

For the maximum load bearing capacity at the tip of the crack, it is observed that almost half the load is required to arrest the crack at this inner tip of the bigger crack as compared to that at the exterior tip ofthe smaller crack.

Also, when comparing the behavior at the interior tips of the bigger and smaller crack it is observed that the interior tip of the smaller crack is most stressed while the interior tip of the bigger crack shows a closing behavior but the variation is comparatively much linear.

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